

Graphing Rational Functions

$$f(x) = \frac{N(x)}{D(x)} \text{ where } N \text{ and } D \text{ are polynomials with no common factors.}$$

I. **V.A.:** Find the vertical asymptote(s) by setting the denominator equal to zero: $D(x) = 0$

II. **H.A.:** Find the horizontal asymptote by using the following rules:

$$f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \dots}{b_m x^m + a_{m-1} x^{m-1} + \dots} \quad \text{The degree of the numerator is } n. \text{ The degree of denominator is } m.$$

a) If $n < m$, then $y=0$ (x-axis) is the horizontal asymptote

b) If $n=m$, then $y = \frac{a_n}{b_m}$ is the horizontal asymptote.

c) If $n > m$, then there is no horizontal asymptote

Exception: If $n > m$ by 1, then the graph has a slant asymptote (S.A.)

III. Find the x-intercepts by setting the numerator equal to zero and solve. $N(x) = 0$

IV. Find the y-intercepts by substituting zero for x in the function, $f(0)$

V. Make a table of values between and beyond each x-intercept and vertical asymptote; *more values provide a more accurate graph.*

VI. Sketch the graph by using the information obtained.

. NOTE: Remember that the graph never crosses the vertical asymptotes!!!!

EXAMPLE 1: *Graph the function -* $f(x) = \frac{3x^2}{x^2 - 4}$

I. **V.A.:** Set the denominator equal to zero and solve.

$$\begin{aligned} x^2 - 4 &= 0 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned} \quad \text{Vertical asymptotes at } x = 2 \text{ \& } x = -2$$

II. **H.A.:** The degree of the numerator and denominator are both equal to 2. So the horizontal asymptote is $y = \frac{3}{1}$.

Horizontal asymptote at $y=3$

III. **x-intercept(s):** Set the numerator equal to zero and solve.

$$\begin{aligned} 3x^2 &= 0 \\ x^2 &= 0 \\ x &= 0 \end{aligned} \quad \text{Intercept at } x = 0$$

IV. **y-intercept(s):** Substitute zero for x in the function and solve.

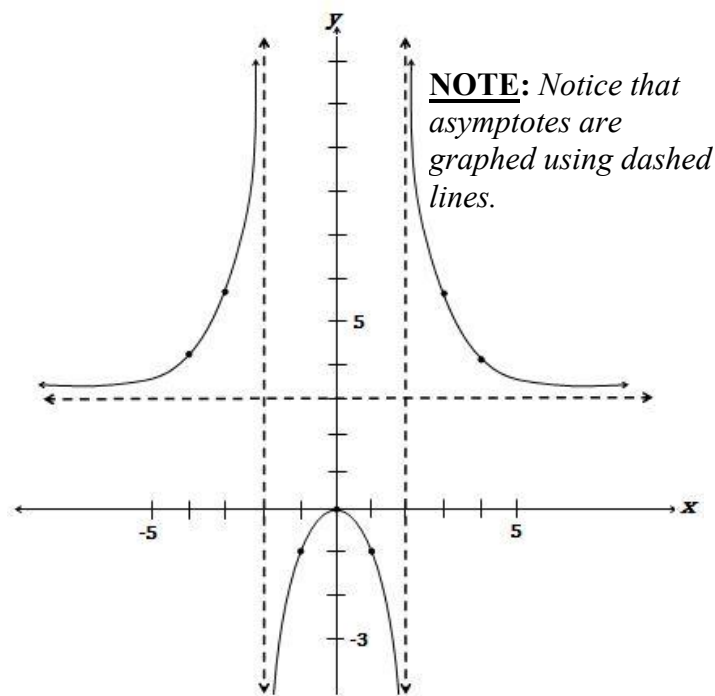
$$y = \frac{3(0)^2}{(0)^2 - 4} = -\frac{0}{4} \quad \text{Intercept at } y = 0$$

$y = 0$

V. **Table:** Make a table of values near each vertical asymptote and x-intercept and plot the values

x	$f(x) = \frac{3x^2}{x^2 - 4}$
-4	4
-3	$\frac{27}{5}$
-1	-1
1	-1
3	$\frac{27}{5}$
4	4

VI. **Graph:** Sketch the graph by using the information obtained.



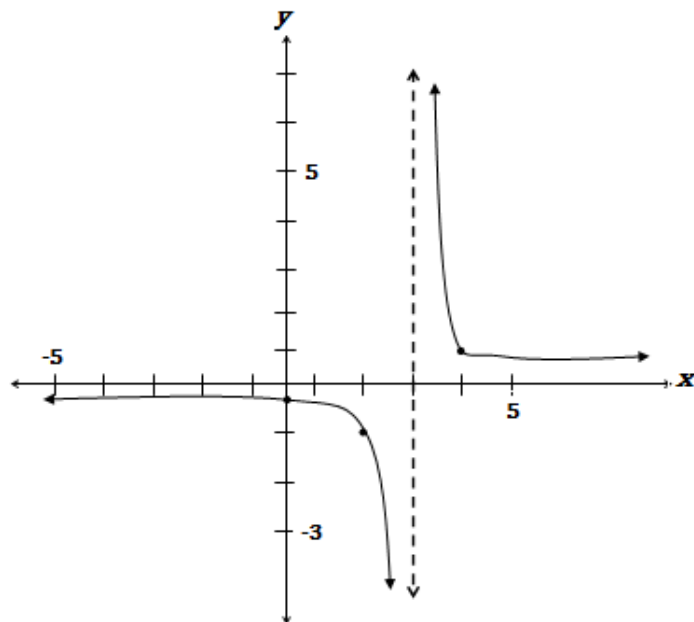
EXAMPLE 2: Graph the function - $f(x) = \frac{x+3}{x^2-9}$

$$f(x) = \frac{x+3}{x^2-9} = \frac{\cancel{x+3}}{(\cancel{x+3})(x-3)} = \frac{1}{(x-3)}$$

VI. **Graph:**

- I. **V.A.:** $x - 3 = 0$ $x = 3$
- II. **H.A.:** $n < m$ so $y = 0$
- III. **x-intercept:** $1 \neq 0$, so there are none
- IV. **y-intercept:** $\frac{1}{0-3} = -\frac{1}{3}$
- V. **Table:**

x	$f(x) = \frac{x+3}{x^2-9}$
2	-1
4	1



EXAMPLE 3: *Graph the function - $f(x) = \frac{x+3}{x^2+9}$*

I. **V.A.:** $x^2 + 9 \neq 0$
The function is always positive, so there are no vertical asymptotes

II. **H.A.:** $n < m$ so $y = 0$

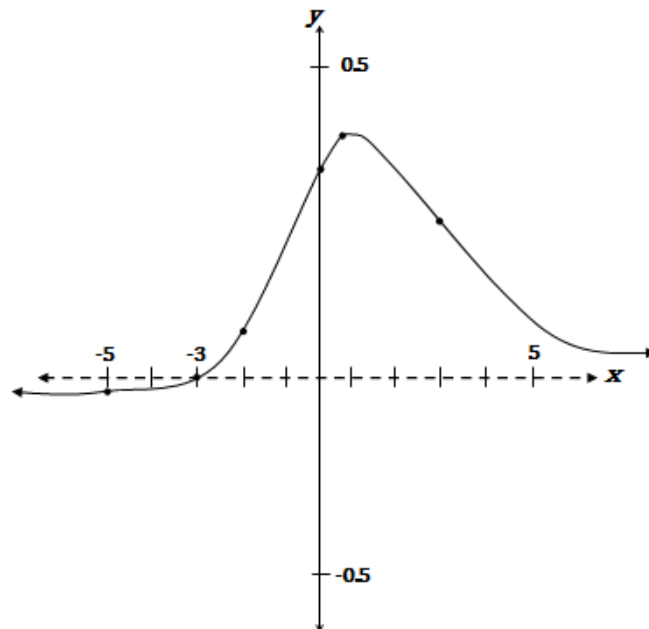
III. **x-intercept:** $x + 3 = 0$ $x = -3$

IV. **y-intercept:** $\frac{0+3}{0^2+9} = \frac{1}{3}$

V. **Table:**

x	-5	-2	1	3
$f(x) = \frac{x+3}{x^2+9}$	$-\frac{1}{17}$	$\frac{1}{13}$	$\frac{2}{5}$	$\frac{1}{3}$

VI. **Graph:**



EXAMPLE 4: *Graph the function - $f(x) = \frac{x^2+1}{x-1}$*

I. **V.A.:** $x - 1 = 0$ $x = 1$

VI. **Graph:**

II. **H.A.:** $n > m$ by 1, therefore there are no horizontal asymptotes, but there is a slant asymptote; the equation can be found by division

$$f(x) = \frac{x^2+1}{x-1} = \boxed{x+1} + \frac{2}{x-1}$$

Equation of Slant Asymptote
 $y = x+1$

III. **x-intercept:** $x^2 + 1 = 0$ $x^2 \neq -1$
so there are none

IV. **y-intercept:** $\frac{0^2+1}{0-1} = -1$

V. **Table:**

x	$-\frac{1}{2}$	2
$f(x) = \frac{x^2+1}{x-1}$	$-\frac{5}{6}$	5

